Lecture 11.1 : The Magnetic Field Continued

Lecture Outline:
Magnetic Field of a Current
Magnetic Dipoles
Ampere’s Law
Magnetic Force on Moving Charge

Textbook Reading:
Ch. 32.4 - 32.7

March 24, 2015
Announcements

• Read Ch. 32

• Homework #8 due on Tuesday, March 31 in Mastering Physics.

• Exam #2 Results: Average 76.40%, Standard Deviation 14.43%
Some Properties of Magnets:

Magnets have two poles (north and south). Like poles repel, opposites attract.

Some materials are attracted to magnets, some are not.

If allowed to rotate, like in a compass, a magnet will align itself in a north-south direction.
Biot-Savart law

\[ \vec{B}_{\text{point charge}} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2} \]

The Cross Product: \( \vec{C} \times \vec{D} = CD \sin \alpha \)

Direction from right-hand rule.
Magnetic Field at a distance $d$ away from a long, straight wire carrying current $I$ (see Example 32.3 for derivation)

$$\vec{B}_{\text{wire}} = \frac{\mu_0 I}{2\pi d}$$

Use right-hand rule to determine direction
Magnetic Field of a Current

\[ B_{\text{loop}} = \frac{\mu_0}{2} \frac{IR^2}{(z^2 + R^2)^{3/2}} \]

See example 32.5 in text for derivation
Magnetic Dipoles

Notice that a current loop has a directionality. Magnetic field flows in one side, and out through the other, just like a magnet.

Whether it’s a current loop or a permanent magnet, the magnetic field emerges from the north pole.

The current loop is a magnet!
Recall the electric dipole...

\[ \vec{p} = q s \]

Dipole moment, points from negative to positive charge.

\[ \vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi \epsilon_0} \frac{\vec{p}}{r^3} \]

Electric field along dipole axis, far from dipole.
Magnetic Dipoles

We can also define a magnetic dipole moment

\[ B_{\text{loop}} = \frac{\mu_0}{2} \frac{IR^2}{(z^2 + R^2)^{3/2}} \]

\[ z \gg R \]

\[ B_{\text{loop}} \approx \frac{\mu_0}{2} \frac{IR^2}{z^3} = \frac{\mu_0}{4\pi} \frac{2(\pi R^2)I}{z^3} = \frac{\mu_0}{4\pi} \frac{2AI}{z^3} \]

\[ \vec{\mu} \equiv AI \]

\[ B_{\text{dipole}} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{z^3} \]

Magnetic dipole moment points from south pole to north pole.
Ampere’s Law

We can use the Biot-Savart law to calculate magnetic field of any distribution of moving charges. In practice, this is difficult (like using Coulomb’s Law to calculate E-field of arbitrary charge distributions).

\[ \sum_{k} \vec{B}_k \cdot \Delta \vec{s}_k \rightarrow \int_{i}^{f} \vec{B} \cdot d\vec{s} \]

Line passing through a magnetic field.

Sum/Integrate B along the line
Ampere’s Law

Calculate the same integral around the circumference of a circle with radius $d$, surrounding a wire carrying current $I$.

The integration path is a circle of radius $d$. The integration starts and stops at the same point.

B is constant/parallel to $ds$ everywhere on circle!

\[ \int \vec{B} \cdot d\vec{s} = B(2\pi d) \]

\[ \vec{B}_{\text{wire}} = \frac{\mu_0 I}{2\pi d} \]

\[ \int \vec{B} \cdot d\vec{s} = \mu_0 I \]
Ampere’s Law

Result from previous slide turns out to be general:

• Independent of the shape of the curve around the current.
• Independent of where the current passes through curve.
• Depends only on the total current passing through the area enclosed by the integration path.

Ampere’s Law

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{through}} \]

Note that the integral here is around a closed loop, and not over a closed surface like in Gauss’s Law.
Ampere’s Law

Must be careful to determine which currents passing through the integration path are positive and which are negative.

Use right-hand rule to determine sign of the current(s).

What is $I_{\text{through}}$ for this loop?
For the path shown,

\[ \oint \vec{B} \cdot d\vec{s} = \]

A. 0.

B. &mu;₀(\(I_1 - I_2\)).

C. &mu;₀(\(I_2 - I_1\)).

D. &mu;₀(\(I_1 + I_2\)).
Solenoids

A solenoid is a helical coil of wire with current $I$ passing through each loop in the coil.
Solenoids

What is the magnetic field inside the solenoid?

Field (North)

Current

\[ B = \frac{\mu_0 NI}{\ell} \]

\[ \n = \frac{N}{\ell} \text{ turn density} \]
Clicker Question #2

Solenoid 2 has twice the diameter, twice the length, and twice as many turns as solenoid 1. How does the field $B_2$ at the center of solenoid 2 compare to $B_1$ at the center of solenoid 1?

A. $B_2 = B_1/4$.  
B. $B_2 = B_1/2$.  
C. $B_2 = B_1$.  
D. $B_2 = 2B_1$.  
E. $B_2 = 4B_1$.  

Same number of turns/length!
Solenoids

Real Solenoids are Very Useful

Magnetic Resonance Imaging
3 Tesla fields are common.
Solenoids

MRIs feature superconducting solenoid that is powerful!

MRI + Magnetic Material = Bad
Example: What is the magnetic field inside and outside of a current-carrying wire of radius $R$?
Magnetic Force on Moving Charge

Magnetic Force also depends on how moving charge’s velocity is oriented relative to a magnetic field.

\[ \vec{F}_{\text{on } q} = q\vec{v} \times \vec{B} = (qvB \sin \alpha, \text{ direction of right-hand rule}) \]

- There is no force on a charge moving parallel to \( \vec{B} \).
- The magnetic force is perpendicular to \( \vec{v} \) and \( \vec{B} \). Its magnitude is \( qvB \sin \alpha \).
- The magnetic force is maximum when the charge moves perpendicular to \( \vec{B} \).
The direction of the magnetic force on the proton is

A. To the right.
B. To the left.
C. Into the screen.
D. Out of the screen.
E. The magnetic force is zero.
Clicker Question #4

Which magnetic field causes the observed force?
Example: What’s the magnetic force on the electron if it’s 1.0cm above a 10A current, and it’s traveling at $1.0\times10^7$ m/s to the right?

Follow-up: What happens to the electron as it travels? Does it speed up or slow down?
• Read Ch. 32
• Homework #8 is due Tuesday (March 31) at 9am.