Please put away everything except a pen/pencil and a calculator (if you brought one).

No talking, and no using phones/computers during Quiz!

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Lecture 14.2:
Maxwell’s Equations

Lecture Outline:
The Displacement Current
Maxwell’s Equations
Electromagnetic Waves

Textbook Reading:
Ch. 34.3 - 34.5

April 18, 2013
• Homework #11 due on Monday, April 22, at 9pm.

• Online Evaluation e-mails will be sent to you on Monday, April 22.
  ‣ Please fill out the evaluation form...it is completely confidential. May 8 is deadline.
  ‣ Remember that PHY212 and PHY222 (lab) are separate courses!

• Society of Physics Student (SPS) is planning to offer “clinic” services to you during Finals week. More details to come.
More generally, a charge can move through E and B fields with velocity $v_{CA}$. We can imagine an observer moving at the same velocity as the charge.

\[
\vec{F}_A = q(\vec{E}_A + \vec{v}_{CA} \times \vec{B}_A)
\]

\[
\vec{E}_B = \vec{E}_A + \vec{v}_{CA} \times \vec{B}_A
\]

Note: Frame B still can’t say anything about possible magnetic fields since they are at rest with respect to the charge.
General equation for transforming electric and magnetic fields between two inertial reference frames:

\[
\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A
\]

\[
\vec{B}_B = \vec{B}_A - \frac{1}{c^2} \vec{v}_{BA} \times \vec{E}_A
\]
Example: Scientists in the laboratory create a uniform electric field \( E = 1.0 \times 10^6 \text{ V/m} \) in the positive z-direction in a region of space where \( B = 0 \text{ T} \). What are the E and B fields in the reference frame of a rocket traveling in the positive x-direction at \( 1.0 \times 10^6 \text{ m/s} \)?
The Displacement Current

Recall Ampere’s Law:

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{through}} \]
No rule saying we have to pick surface $S_1$. We could also choose to evaluate Ampere’s Law through $S_2$, which is bounded by the same curve.
Consider the circuit below with a capacitor and a battery. It appears we will get very different values for $I_{\text{through}}$ depending on which surface we use.

The magnetic field of the current $I$ that is charging the capacitor.

$$\int \vec{B} \cdot d\vec{s} = \mu_0 \left( I_{\text{through}} + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

The flux through capacitor plates:

$$\Phi_E = EA$$

The electric field of the capacitor:

$$E_{\text{capacitor}} = \frac{Q}{\varepsilon_0 A}$$

Flux through area $A$:

$$\Phi_E = \frac{Q}{\varepsilon_0 A} \cdot A = \frac{Q}{\varepsilon_0}$$

$$\frac{d\Phi_E}{dt} = \frac{I}{\varepsilon_0} \frac{dQ}{dt} = \frac{I}{\varepsilon_0}$$

Displacement current:

$$I_{\text{displacement}} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \left( I_{\text{through}} + I_{\text{displacement}} \right)$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \left( I_{\text{through}} + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$
The Displacement Current

\[ \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \]

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 \left( I_{\text{through}} + \epsilon_0 \frac{d\Phi_e}{dt} \right) \]
Maxwell’s Equations

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \quad \text{Gauss’s law} \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \quad \text{Gauss’s law for magnetism} \]

\[ \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_m}{dt} \quad \text{Faraday’s law} \]

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{through}} + \varepsilon_0 \mu_0 \frac{d\Phi_e}{dt} \quad \text{Ampère-Maxwell law} \]

\[ \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \text{(Lorentz force law)} \]
Maxwell’s Equations

No Isolated Magnetic Poles (Monopoles)!

\((\Phi_m)_{\text{closed surface}} = \oint \vec{B} \cdot d\vec{A} = 0\)
**Electromagnetic Waves**

1. A sinusoidal wave with frequency $f$ and wavelength $\lambda$ travels with wave speed $v_{\text{em}}$.

2. $\vec{E}$ and $\vec{B}$ are perpendicular to each other and to the direction of travel. The fields have amplitudes $E_0$ and $B_0$.

3. $\vec{E}$ and $\vec{B}$ are in phase. That is, they have matching crests, troughs, and zeros.

“Source-free” Maxwell’s equations (no charge or current)

\[
\oint \vec{E} \cdot d\vec{A} = 0
\]

\[
\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}
\]

\[
\oint \vec{B} \cdot d\vec{A} = 0
\]

\[
\oint \vec{B} \cdot d\vec{s} = \epsilon_0 \mu_0 \frac{d\Phi_e}{dt}
\]
Electromagnetic Waves

\[ E_y = E_0 \sin \left(2\pi \left(\frac{x}{\lambda} - ft\right)\right) \]

\[ B_z = B_0 \sin \left(2\pi \left(\frac{x}{\lambda} - ft\right)\right) \]

\[ E_x = E_z = 0 \]

\[ B_x = B_y = 0 \]
Electromagnetic Waves

\[ \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \]

Condition on EM Waves from Faraday’s Law

\[ E_y = E_0 \sin(2\pi \left( \frac{x}{\lambda} - ft \right)) \quad B_z = B_0 \sin(2\pi \left( \frac{x}{\lambda} - ft \right)) \]

\[ \frac{\partial E_y}{\partial x} = \frac{2\pi E_0}{\lambda} \cos(2\pi \left( \frac{x}{\lambda} - ft \right)) \quad \frac{\partial B_z}{\partial t} = \left(-2\pi f B_0 \cos(2\pi \left( \frac{x}{\lambda} - ft \right)) \right) \]

\[ \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad \text{requirement for EM waves from Faraday’s Law.} \]

\[ \frac{2\pi E_0}{\lambda} \cos(2\pi \left( \frac{x}{\lambda} - ft \right)) = 2\pi f B_0 \cos(2\pi \left( \frac{x}{\lambda} - ft \right)) \]

\[ \frac{E_0}{\lambda} = f B_0 \]

\[ E_0 = \lambda f B_0 = v_{em} B_0 \]

\[ v_{em} = \lambda f \]

\[ v_{em} < c \quad \text{for EM waves.} \]

\[ E_0 = v_{em} B_0 \]
Electromagnetic Waves

The Electromagnetic Spectrum

<table>
<thead>
<tr>
<th>Wavelength (meters)</th>
<th>Radio $10^3$</th>
<th>Microwave $10^{-2}$</th>
<th>Infrared $10^{-5}$</th>
<th>Visible $.5 \times 10^{-6}$</th>
<th>Ultraviolet $10^{-8}$</th>
<th>X-ray $10^{-10}$</th>
<th>Gamma Ray $10^{-12}$</th>
</tr>
</thead>
</table>

About the size of...

- Buildings
- Humans
- Honey Bee
- Pinpoint
- Protozoans
- Molecules
- Atoms
- Atomic Nuclei

Frequency (Hz)

$10^4$ $10^8$ $10^{12}$ $10^{15}$ $10^{16}$ $10^{18}$ $10^{20}$
• Read Ch. 34.
• Quiz #5 on Thursday.