Lecture 5.2 : Electric Potential and Field

Lecture Outline:
Electric Potential and Field
Sources of Electric Potential
Finding Field from Potential

Textbook Reading:
Ch. 28.7 - 29.3

Feb. 14, 2013
Announcements

- HW5 due next Mon. (2/18) at 9pm on Mastering Physics.
- Quiz #3 next Thu. (Feb. 21) in class.
Two protons, one after the other, are launched from point 1 with the same speed. They follow the two trajectories shown. The protons’ speeds at points 2 and 3 are related by

A. \( v_2 > v_3 \).

B. \( v_2 = v_3 \).

C. \( v_2 < v_3 \).

D. Not enough information to compare their speeds.

NOTE: This answer can be seen most easily if you use Energy Conservation arguments. If you use kinematic arguments, be careful to note that the two trajectories don’t take equal time!
Equipotential Surfaces are surfaces with the same value of V at every point.
3D Map of Potential around a positive charge.

\[ V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} \]

3D Map of Potential around a dipole.

\[ V = \sum_i \frac{1}{4\pi \varepsilon_0} \frac{q_i}{r_i} \]
We’ve now introduced several different concepts:

Electric Potential and Field

We've now introduced several different concepts:

\[ V \equiv \frac{U_{q+\text{sources}}}{q} \]

- Force concept (Acts locally): \( \vec{F} \)
- Energy concept (Everywhere in space): \( U \)

\[ \vec{E}(x, y, z) = \frac{\vec{F}_{\text{on } q \text{ at } (x, y, z)}}{q} \]
Electric Potential and Field

Let’s connect the other sides...

\[ \Delta U = -W(i \rightarrow f) = - \int_{s_i}^{s_f} F_s ds = - \int_i^f \vec{F} \cdot d\vec{s} \]

\[ \Delta V = V_f - V_i = - \int_{s_i}^{s_f} E_s ds = - \int_i^f \vec{E} \cdot d\vec{s} \]
Electric Potential and Field

\[ \Delta U = - \int_i^f \vec{F} \cdot d\vec{s} \]

\[ \Delta V = - \int_i^f \vec{E} \cdot d\vec{s} \]

Act locally: \( \vec{F} \)

Everywhere in space: \( \vec{E} \)

Energy concept:

\[ V \equiv \frac{U_{q+\text{sources}}}{q} \]

Force concept:
Clicker Question #1

This is a graph of the $x$-component of the electric field along the $x$-axis. The potential is zero at the origin. What is the potential at $x = 1\text{m}$?

A. 2000 V.
B. 1000 V.
C. 0 V.
D. -1000 V.
E. -2000 V.
Let’s apply these relations to a single point charge:

\[ \Delta V = V(r) - V(\infty) = \int_{r}^{\infty} E_s \, ds = \int_{r}^{\infty} E_s \, ds \]

\[ E_s = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{s^2} \]

\[ V(r) = V(\infty) + \frac{q}{4\pi\varepsilon_0} \int_{r}^{\infty} \frac{ds}{s^2} = V(\infty) + \frac{q}{4\pi\varepsilon_0} \left( \frac{-1}{s} \right) \bigg|_{r}^{\infty} \]

\[ = 0 + \frac{q}{4\pi\varepsilon_0} \left( \frac{-1}{\infty} - \frac{-1}{r} \right) \]

\[ = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \]
Now let’s apply these relations to a capacitor:
Sources of Electric Potential

Van de Graaff Generator

2. The plastic or leather belt is the conveyor belt that mechanically transports charge to the top.

3. A pointed wire draws charge off the belt and charges the sphere.

1. A corona discharge charges the belt positively.
Sources of Electric Potential

Batteries
Sources of Electric Potential

Batteries

Batteries arranged in **series**

\[ \Delta V_{\text{series}} = \Delta V_1 + \Delta V_2 + \cdots \]

The charge escalator “lifts” charge from the negative side to the positive side. Charge \( q \) gains energy \( \Delta U = q\Delta V_{\text{bat}} \).
Sources of Electric Potential

emf ("E"-"M"-"F")

\[ \Delta V_{\text{bat}} = \frac{W_{\text{chem}}}{q} = \mathcal{E} \]

Example Problem: A battery does \(4.8\times10^{-19}\) J of work on a proton that it moves from the negative to positive terminal. What is the battery’s emf?

\[ \text{emf} = \frac{W}{q} = \frac{(4.8\times10^{-19} \text{ J})}{(1.6\times10^{-19} \text{ C})} = 3 \text{ V} \]
If we move a distance $ds$ through a potential difference of $dV$, the component of Electric Field in the $s$-direction is:

$$E_s = -\frac{dV}{ds}$$

$E$ is perpendicular to equipotential surfaces and points “downhill” in the direction of decreasing potential.
At which point is the electric field strongest (i.e. - largest magnitude)?

A. At $x_A$.
B. At $x_B$.
C. The field is the same strength at both.
D. There’s not enough information to tell.
An electron is released from rest at $x = 2 \text{ m}$ in the potential shown. What does the electron do immediately after being released?

A. Stay at $x = 2 \text{ m}$.
B. Move to the right ($+ x$) at steady speed.
C. Move to the right with increasing speed.
D. Move to the left ($- x$) at steady speed.
E. Move to the left with increasing speed.

Slope of $V$ negative => $E_x$ is positive (field to the right). Electron is negative => force to the left. Force to the left => acceleration to the left.
More generally:

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k} = - \left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)$$

$$\vec{E} = -\nabla V$$

Example: The electric potential along the y-axis is: $V = 50y^3$ Volts, where $y$ is in meters. What is $E_y$ at $y=1m$?
Finding Field from Potential

1. $\vec{E}$ is everywhere perpendicular to the equipotential surfaces.

2. $\vec{E}$ points “downhill,” in the direction of decreasing $V$.

3. The field strength is inversely proportional to the spacing $\Delta s$ between the equipotential surfaces.

4. Equipotential surfaces have equal potential differences between them.
Kirchhoff’s Loop Law

$$\Delta V_{\text{loop}} = \sum_i (\Delta V)_i = 0$$
Reminders

• Show up for recitations! Problems assigned during recitations are fair game for quizzes and exams.

• Quiz #3 next Thursday (Feb. 21)

• Read Ch. 29